

Mathematics II

(English course)

Second semester, 2012/2013

Exercises (2)

1. Classify the following quadratic forms:

$$\begin{aligned}Q_1(x, y) &= 4x^2 + 4xy + y^2, & Q_2(x_1, x_2) &= x_1x_2, \\Q_3(x_1, x_2) &= x_1^2 + 2x_1x_2 - x_2^2, & Q_4(x, y) &= 34x^2 - 24xy + 41y^2, \\Q_5(x_1, x_2, x_3) &= x_1^2 + x_1x_2 + x_1x_3 + x_2x_3, \\Q_6(y, y, z) &= 2x^2 + 4xz + y^2 - z^2, \\Q_7(x, y, z) &= 3x^2 + 4xy + 8xz + 4yz + 3z^2.\end{aligned}$$

2. For each of the quadratic forms above, find the associated symmetric matrix and find the eigenvalues of that matrix.
3. Find all the real numbers t making the quadratic form

$$Q_t(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 2tx_1x_2 + 2x_1x_3$$

positive definite.

4. Classify the quadratic forms represented by the following matrices:

$$\begin{aligned}A &= \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}, & B &= \begin{bmatrix} 0 & -2 \\ -2 & 10 \end{bmatrix}, & C &= \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, \\D &= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & 5 \end{bmatrix}, & E &= \begin{bmatrix} 2 & 3 & -4 \\ 3 & 1 & 2 \\ -4 & 2 & 3 \end{bmatrix}, & F &= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \\G &= \begin{bmatrix} -2 & 1 & -2 \\ 1 & -1 & 1 \\ -2 & 1 & -7 \end{bmatrix}, & H &= \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}, & J &= \begin{bmatrix} 2 & 0 & 8 \\ 2 & 2 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \\K &= \begin{bmatrix} 2 & -3 \\ -3 & a \end{bmatrix}, & L &= \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ a & 0 & 0 & 3 \end{bmatrix}, & M &= \begin{bmatrix} a & 0 & 0 \\ 0 & 5 & a \\ 0 & 0 & 2 \end{bmatrix}.\end{aligned}$$